

Heavy-light mesons from the AdS/CFT correspondence

Johanna Erdmenger and Johannes Große

*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 München, Germany
E-mail: jke@mppmu.mpg.de, jgrosse@mppmu.mpg.de*

Nick Evans

*School of Physics & Astronomy, Southampton University
Southampton, S017 1BJ, United Kingdom
E-mail: evans@phys.soton.ac.uk*

ABSTRACT: We propose a holographic description of heavy-light mesons, i.e. of mesons containing a light and a heavy quark. In the semi-classical string limit, we look at the dynamics of strings tied between two D7 branes. We consider this setup both in an AdS background and in the non-supersymmetric Constable-Myers geometry which induces chiral symmetry breaking. We compute the meson masses in each case. Finally we discuss the results relevance for phenomenological comparison to the physical b-quark sector.

KEYWORDS: AdS-CFT Correspondence, D-branes.

Contents

1.	Introduction	1
2.	Mesons in AdS	3
2.1	Probe D7 branes in AdS	3
2.2	Heavy-light mesons in AdS	5
3.	Heavy-light mesons in dilaton flow geometry	7
3.1	Dilaton flow geometry	7
3.2	Heavy-light mesons	11
4.	Bottom phenomenology	13

1. Introduction

The AdS/CFT correspondence [1–3] and its generalizations offer the possibility to holographically describe the strong coupling regime of QCD using weakly coupled gravity. There has been steady progress towards such a description including string theory descriptions of theories that break chiral symmetry [4] – [23]. Most recently these ideas have been adapted to produce phenomenological models of QCD dynamics that describe aspects of the meson and baryon spectrum at the 20% level or better [24] – [43].

Within models based on AdS/CFT, quarks may be introduced by the inclusion of D7 branes in the holographic space [44] – [52].¹ A small number of quarks, ψ^i , ($N_f \ll N_c$), may be described by treating the D7 branes as probes [44, 47]. In the simplest case with the background geometry $\text{AdS}_5 \times S^5$, the geometry around a stack of D3 branes, the resulting field theory is an $\mathcal{N} = 2$ gauge theory with fundamental hypermultiplets. On the field theory side of the correspondence, the quarks are described by strings stretching between the D3 and the D7 branes. The quark mass is proportional to the separation between the D3 brane stack and the D7 probe. The holographic dual description of the quark bilinear $\bar{\psi}\psi$ is given by open strings with both ends on the D7 brane probe in the $\text{AdS}_5 \times S^5$ background. Therefore fluctuations of the D7 brane correspond to meson excitations. In [4], a holographic description of chiral symmetry breaking and light mesons was given by embedding a D7 brane probe into a deformed non-supersymmetric supergravity background.

Here we push these ideas into a new arena and describe mesons made of one light quark, ψ_L , which in QCD would experience chiral symmetry breaking dynamics, and one heavy quark, ψ_H , that would not. To begin with, we formulate our new approach by considering

¹Related models involving other brane setups may be found in [53] – [62].

a string theory description of the supersymmetric field theory dual to $\text{AdS}_5 \times \text{S}^5$ with D7 probes wrapping $\text{AdS}_5 \times \text{S}^3$, which is conformal when the quarks are massless. To include a heavy quark and a light quark, we must have *two* D7 branes with very different separations from the central D3 brane stack. Heavy-light mesons are described by the strings stretched between these two D7 branes. In the limit where one quark is very heavy, these strings become long, and we may use a semi-classical description of their dynamics. As the simplest ansatz, we just consider the motion of a rigid string in directions transverse to the separation of the D7 branes, i.e. in the x directions of the field theory on the D3 brane as well as in the radial holographic direction. In other words we treat the D7-D7' string as rigidly tied between the two D7 and neither let it bend nor oscillate in the direction of its length. We can then integrate over the string length in the string (Polyakov) action. In this way we obtain an effective point-particle-like action for the string's motion. In the spirit of second quantization, we convert the action to a field equation which we consider as the holographic field theory description of the heavy-light mesons. From the field-theory point of view, this equation is a generalization of the Klein-Gordon equation.² Note that in our semi-classical string theory formalism, vector and scalar meson masses are inherently degenerate. On the field theory side, this ties in well with the fact that spin effects are suppressed by $1/m_H$ with m_H the heavy quark mass [70].

Note that the field theory describing the heavy-light mesons does not exist in the same space-time as the fields holographically describing light-light or heavy-heavy mesons, but in a space given by the integrated average over the space between the two D7. For this reason one needs a fully stringy picture of the D7 branes to describe these states and they cannot be extracted from the simple five-dimensional models found in [24] – [43].

In this manner we present results for the heavy-light states in the $\mathcal{N} = 2$ supersymmetric gauge theory so far described. However this model does not include chiral symmetry breaking behaviour. Therefore we move on to the authors' preferred string theory description of chiral symmetry breaking [4] in the non-supersymmetric Constable-Myers background [71 – 74]. This model consists of an AdS background deformed by the presence of a non-zero dilaton. The presence of the dilaton breaks the conformal and supersymmetries of the string set-up, leaving a non-supersymmetric strongly coupled confining gauge background. The advantage of this background is that in the UV, it returns to four-dimensional $\mathcal{N} = 2$ theory (with the field content of $\mathcal{N} = 4$ plus fundamental matter), such that the asymptotic behaviour of the brane embedding is well under control.

Quarks are again introduced by D7 brane probes in the deformed geometry. It is known from [4] that the D7 branes are repelled from the center of the geometry, triggering the formation of a chiral symmetry breaking condensate. In order to obtain heavy-light mesons and their masses, we use our new formalism developed in the supersymmetric case. This time however, for the deformed background, we have to integrate numerically over the length of D7-D7' string.

Phenomenological holographic models have taught us that such string theory models

²In a somewhat different approach which uses spinning strings, multiflavour mesons were also considered in [63]. Further related work may be found in [64] – [69].

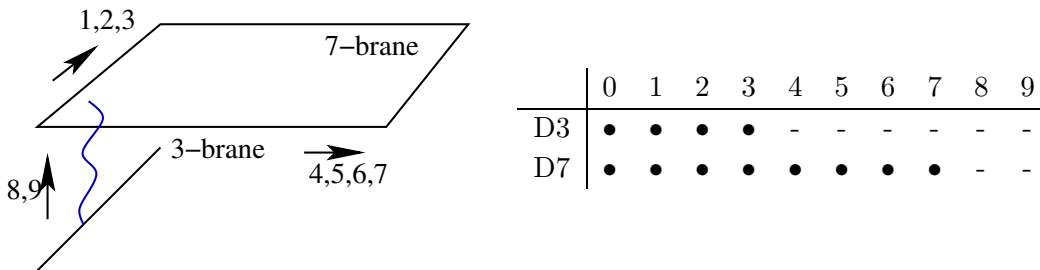


Figure 1: The basic geometry of the D3-D7 system under consideration.

work well at describing QCD. We therefore allow ourselves to be tempted into describing the QCD b-quark mesons, even though the b-quark is not infinitely massive and the gauge background is not that of QCD. The AdS description, which is fully valid only at infinite 't Hooft coupling λ , predicts that the light-light and heavy-heavy meson masses are suppressed relative to the constituent quark mass by $\sqrt{\lambda}$. There is relatively little evidence for this behaviour in real QCD. However, in this limit we find that the mass of the lowest of the heavy-light states coincides with the heavy quark mass. We use the ρ , Υ and B meson masses to fix Λ_{QCD} , the heavy quark mass m_H and λ in our model. The ρ and Υ mesons are light-light and heavy-heavy vector mesons, respectively. This procedure generates the result $\lambda \sim 5$ and allows for a prediction of the B^* meson mass, which however yields a result which is 20% too high. The observed $B - B^*$ splitting can only be generated with a much larger value of λ which makes the ρ, Υ too light. The model does provide a rough caricature of the QCD states though.

2. Mesons in AdS

2.1 Probe D7 branes in AdS

Quark fields may be introduced into the $\mathcal{N} = 4$ gauge theory described by the AdS/CFT correspondence by including probe D7 branes. The resulting theory is an $\mathcal{N} = 2$ theory investigated previously in [44, 47]. Neglecting the back reaction of the D7 in the probe limit corresponds to considering the quenched approximation on the field theory side. As shown in figure 1, strings which stretch between the D3 and D7 branes are in the fundamental representation of the $SU(N)$ gauge theory on the D3. The length of the minimum length string between the two branes determines the mass of the quark field.

The gravity dual of the D3 branes is $\text{AdS}_5 \times S^5$, in which the D7 brane probe wraps $\text{AdS}_5 \times S^3$. This configuration minimizes the world-volume action of the D7 probe.

The AdS metric is usually written as

$$ds^2 = \frac{w^2}{R^2} \eta_{\alpha\beta} dx^\alpha dx^\beta + \frac{R^2}{w^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dw_5^2 + dw_6^2), \quad w^2 = \rho^2 + w_5^2 + w_6^2, \quad (2.1)$$

where Ω_3 corresponds to a three-sphere and $R^4 = 4\pi g_s N \alpha'^2$. Note that scale transformations in the field theory, which define the mass dimension of operators (for example if we

scale $x \rightarrow e^h x$ then a scalar field of dimension one scales as $\phi \rightarrow e^{-h} \phi$, are mapped to a symmetry of the metric with the radial direction, w , scaling as an energy scale.

We place the D7 brane so that its world-volume coordinates are $\xi^a = x^\alpha, \rho, \Omega_3$. Strings with both ends on the D7 brane generate the DBI action for the brane

$$S_{D7} = -T_7 \int d^8 \xi e^\phi [-\det(\mathbf{P}[g_{ab}])]^{\frac{1}{2}}, \tag{2.2}$$

where the pull-back of the metric $\mathbf{P}[g_{ab}]$ is given by

$$\mathbf{P}[g_{ab}] = g_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}. \tag{2.3}$$

In $AdS_5 \times S^5$ this gives

$$S_{D7} = -T_7 \int d^8 \xi \epsilon_3 \rho^3 \sqrt{1 + \frac{R^2 g^{ab}}{\rho^2 + w_5^2 + w_6^2} (\partial_a w_5 \partial_b w_5 + \partial_a w_6 \partial_b w_6)}, \tag{2.4}$$

where g_{ab} is the induced metric on the D7 and ϵ_3 is the determinant factor from the 3-sphere. Moreover $T_7 = (2\pi^7 g_s \alpha'^4)^{-1}$.

The regular D7 brane embedding solutions are just $w_5^2 + w_6^2 = d^2$, i.e. the brane lies at a constant radius in the $w_5 - w_6$ plane with $m = d/(2\pi\alpha')$ the quark mass. d is the length of the shortest D3-D7 string.

Fluctuations of the D7 brane (which asymptotically fall off as $1/\rho^2$) are dual to meson fields made of the fermionic quark and anti-quark. If we work in the particular choice of background embedding $w_5 = 0, w_6 = d$ and parametrize fluctuations as

$$w_6 + iw_5 = d + \delta(\rho) e^{ik \cdot x}, \quad M^2 = -k^2, \tag{2.5}$$

then the linearized equation of motion for δ is

$$\partial_\rho^2 \delta(\varrho) + \frac{3}{\varrho} \partial_\varrho \delta(\varrho) + \frac{\bar{M}^2}{(\varrho^2 + 1)^2} \delta(\varrho) = 0, \quad \varrho \equiv \frac{\rho}{d}, \quad \bar{M}^2 \equiv -\frac{k^2 R^4}{d^2}. \tag{2.6}$$

The solution is given in terms of hypergeometric functions [47],

$$\delta(\rho) = \frac{1}{(\rho^2 + d^2)^{n+1}} F(-n-1, -n; 2, -\rho^2/d^2). \tag{2.7}$$

The mass spectrum of the degenerate scalar and pseudo-scalar states is then given by

$$M = \frac{2d}{R^2} \sqrt{(n+1)(n+2)}, \quad n = 0, 1, 2, \dots \tag{2.8}$$

Note that the masses of states scale as $d/R^2 = m(2\pi\alpha')/R^2$, which is linear in the quark mass, independent of α' and scales as one over the square root of the 't Hooft coupling (since $R^4/(2\pi\alpha')^2 = g_s N/\pi$). In the limit $d = 0$ the theory becomes conformal and there is not a discrete spectrum.

If we have two D7 branes embedded at $w_6 = d, D$ respectively then there are meson states of the form “light-light” or “heavy-heavy” with the ratio of masses of the lightest two mesons just d/D .

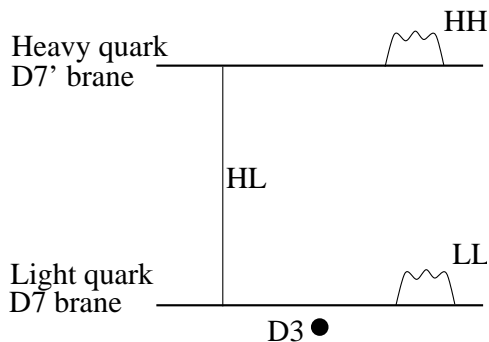


Figure 2: The brane configuration including both a heavy and a light quark. The 77 and $7'7'$ strings are holographic to light-light and heavy-heavy mesons respectively. Heavy-light mesons are holographically described by strings between the two D7 branes — we work in the semi-classical limit where those strings are stretched tight.

2.2 Heavy-light mesons in AdS

We describe mesons made of one heavy and one light quark by strings stretched between the two D7 branes. In the limit $D \gg d$, with D, d the distances of the heavy and the light quark brane from the D3's, these strings are very long such that it is appropriate to consider them in the semi-classical limit (see figure 2).

All of the fields that make up the (non-abelian) DBI action of the D7 branes come from dimensional reduction of ten-dimensional gauge fields. These gauge fields originate from the lightest quantum state of the open strings connecting the D7 branes. Strings with both ends on the inner D7 describe the light quark operators $\bar{\psi}_L \psi_L, \bar{\psi}_L \gamma_5 \psi_L, \bar{\psi}_L \gamma^\mu \psi_L$. Similarly the strings with both ends on the outer D7 describe the heavy quark states. In the scenario proposed here, the strings stretched between the two D7 branes describe the heavy-light modes $\bar{\psi}_H \psi_L, \bar{\psi}_H \gamma_5 \psi_L$ and $\bar{\psi}_H \gamma^\mu \psi_L$. Strings of this type without surface oscillations are the tachyons projected out by the GSO projection. In the semi-classical limit we employ, the string mass is assumed to be dominated by the string length and the subtleties of quantum corrections and surface fluctuations are ignored. The unexcited string stretched between the two D7 branes which we study is therefore an approximation to the scalar, pseudo-scalar and vector heavy-light states which by assumption are degenerate.

We use the gauge-fixed Polyakov string action

$$S_P = -\frac{T}{2} \int d\sigma d\tau G_{\mu\nu} (-\dot{X}^\mu \dot{X}^\nu + X'^\mu X'^\nu), \quad T = \frac{1}{2\pi\alpha'}, \quad (2.9)$$

so we must also impose the constraint equations

$$G_{\mu\nu} \dot{X}^\mu X'^\nu = 0, \quad G_{\mu\nu} (\dot{X}^\mu \dot{X}^\nu + X'^\mu X'^\nu) = 0. \quad (2.10)$$

Our string will lie between the two D7 branes so that $\sigma = w_6$. We will allow the string to move in the world volume of the D7 brane. We thus integrate over the w_6 direction in order to generate an effective point particle action. With $X^\mu = \{x^\alpha, w^i, w_6\}$ (and $w_5 = 0$)

we find, for massless light quark such that $d = 0$,

$$S_P = -\frac{T}{2} \int d\tau \int_0^D d\sigma \left[-\frac{\rho^2 + \sigma^2}{R^2} \dot{x}^\alpha \dot{x}_\alpha - \frac{R^2}{(\rho^2 + \sigma^2)} \dot{w}^i \dot{w}_i + \frac{R^2}{(\rho^2 + \sigma^2)} \right]. \quad (2.11)$$

Here we have used the background AdS metric in the conventions of (2.1) with $\rho^2 = w^i w_i$, $i = 1, 2, 3, 4$. Integrating (2.11) over σ gives

$$S_P = -\frac{T}{2} \int d\tau [-f(\rho) \dot{x}^2 - g(\rho) \dot{w}_i^2 + g(\rho)], \quad (2.12)$$

where

$$f(\rho) = R^{-2}[\rho^2 D + \frac{1}{3} D^3], \quad g(\rho) = \frac{R^2}{\rho} \arctan(D/\rho). \quad (2.13)$$

We wish to convert this classical dynamics to a second quantised field that will play the role of the holographic field to the heavy-light quark operators. We need the modified energy momentum relation for the particle. That relation is provided by the constraint equations. Firstly note that

$$\dot{X}^\mu X'_\mu = 0 \quad (2.14)$$

is trivially satisfied because the string is not allowed to move in the w_6 direction. The remaining constraint, when integrated over σ , gives

$$\frac{1}{f(\rho)} p_x^2 + \frac{1}{g(\rho)} p_w^2 + T^2 g(\rho) = 0, \quad (2.15)$$

where $p_x^\alpha \equiv \delta\mathcal{L}/\delta\dot{x}_\alpha$, $p_w^i \equiv \delta\mathcal{L}/\delta\dot{w}_i$. Note that (2.15) is a simple modification of the usual $E^2 - p^2 = m^2$ with the effective mass depending on the ρ position of the string. It is worth stressing the form of this energy-momentum relation at large radius ρ . Expanding (2.13), we see that in the UV limit, fg becomes a constant while f/g blows up as ρ^4 . In this limit, (2.15) becomes

$$p_x^2 + \frac{\rho^4}{R^4} p_w^2 + T^2 D^2 = 0. \quad (2.16)$$

$T^2 D^2$ is just the heavy quark mass squared. Note that for motion in the x directions the string behaves, as one might naively expect, as a massive string of mass m_H . However for motion in the holographic w directions the mass of the string is ρ dependent, and the string is effectively massless at large ρ . The form of this asymptotic equation is easily understood in terms of the field theory dilutions (see our discussion under (2.1) above) – the w_i (as well as ρ) scale as field theory energies, such that the canonical momenta p_w scale as a length from the point of view of the field theory. Thus the factor of ρ^4 in must be present in (2.16) to match the dimension of p_x^2 . (Note that R does not scale under four-dimensional conformal transformations.)

Making the usual quantum mechanical operator substitutions, we arrive at the field equation

$$\left[\square_x^2 + \frac{f(\rho)}{g(\rho)} \nabla_w^2 - T^2 f(\rho) g(\rho) \right] \phi = 0. \quad (2.17)$$

This is a modified Klein-Gordon equation. In the UV limit $\rho \rightarrow \infty$, we have

$$\nabla_w^2 \phi = 0. \tag{2.18}$$

of the form

$$\phi = m_{\text{HL}} + \frac{c_{\text{HL}}}{\rho^2} + \dots \tag{2.19}$$

This is the correct form to describe the source and VEV of the heavy-light operator $\psi_{\bar{H}}\psi_L$.

Since there is no heavy-light mass mixing term and no heavy-light bilinear VEV in QCD, the correct background configuration has $m_{\text{HL}}, c_{\text{HL}} = 0$ and we look at linearized fluctuations of the form

$$\phi = h(\rho)e^{ik \cdot x}, \quad M^2 = -k^2, \tag{2.20}$$

where the large ρ behaviour of h is $1/\rho^2$. We substitute the ansatz (2.20) into (2.17) and search numerically for regular solutions. It is most convenient to do so in rescaled coordinates $\varrho = \rho/D$, such that equation (2.17) for our ansatz takes the form

$$\left\{ \frac{\pi}{\lambda} \frac{\varrho^3 + \frac{\varrho}{3}}{\arctan \frac{1}{\varrho}} \nabla_{\varrho}^2 + \left[\varrho + \frac{1}{3\varrho} \right] \arctan \frac{1}{\varrho} + \bar{M}^2 \right\} h(\varrho) = 0, \quad \bar{M}^2 = \frac{M^2}{m_H^2}. \tag{2.21}$$

By finding solutions for which h is regular, this equation allows us to calculate the ratio M/m_H as a function of the 't Hooft coupling λ . Note that from the standard AdS/CFT relation $R^4 = 4\pi g_s N \alpha'^2$ we have $R^2 T = \sqrt{\lambda/\pi}$. The results for the masses of the meson and its excited states are shown in figures 3 and 4. We read off that $M_{\text{HL}}/m_H = 1 + \text{const}/\sqrt{\lambda} + \mathcal{O}(\lambda^{-1})$. We see that in the large λ limit we have $M_{\text{HL}} = m_H$ (this follows since in this limit the ϱ derivative in (2.21) is suppressed). This is in agreement with the naive expectation that the meson mass should be equal to string length times its tension.

For comparison we also plot the meson mass dependence on the 't Hooft coupling for small values of λ in figure 4. Note however that the supergravity approximation is unreliable in this limit.

3. Heavy-light mesons in dilaton flow geometry

3.1 Dilaton flow geometry

The AdS geometry studied so far does not contain the physics of most interest in the heavy-light sector of QCD. The main interest lies in the case where the light quark is involved in chiral symmetry breaking and has a dynamically generated mass whilst the heavy quark does not. Chiral symmetry breaking is forbidden in a supersymmetric theory so we will turn instead to a non-supersymmetric dilaton flow geometry that provides a gravity dual of QCD [4].

That model consists of a deformed AdS geometry [71] – [74]

$$ds^2 = H^{-1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^6 dw_i^2, \tag{3.1}$$

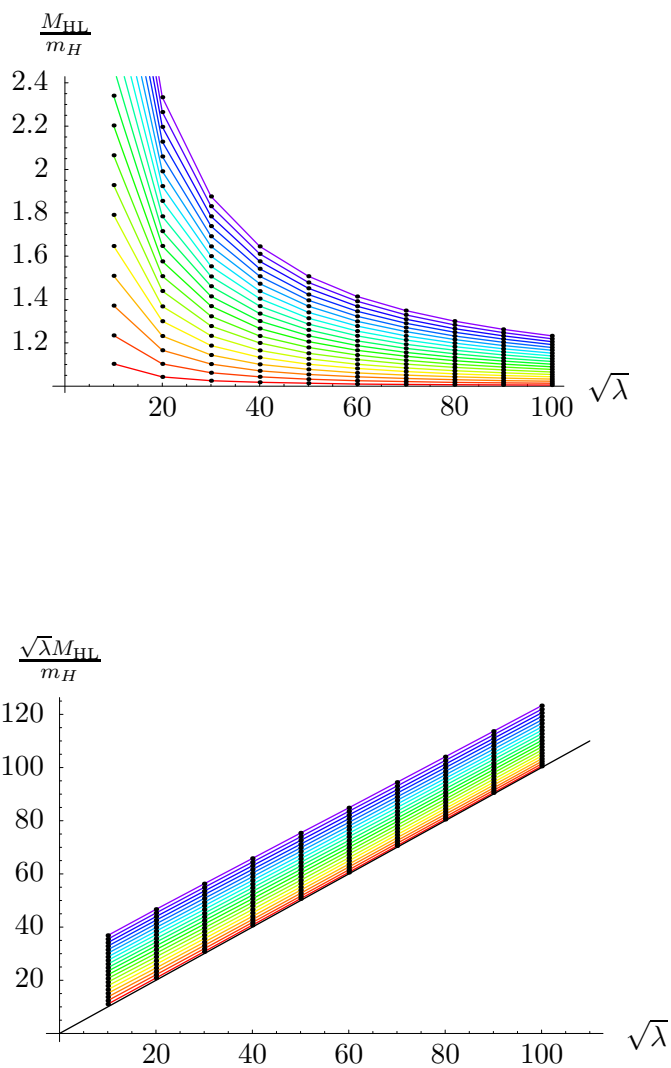


Figure 3: We display the meson and its excited states' masses, M_{HL} , in AdS. The ratio M_{HL}/m_H , with m_H the heavy quark mass (the light quark is taken to be massless), is plotted against the square root of the 'tHooft coupling, λ . We observe that in the large λ limit, M_{HL}/m_H behaves as $1 + \text{const}/\sqrt{\lambda} + \mathcal{O}(\lambda^{-1})$. The black line in the second plot corresponds to $M_{HL}/m_H = 1$.

where

$$H = \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^\delta - 1 \tag{3.2}$$

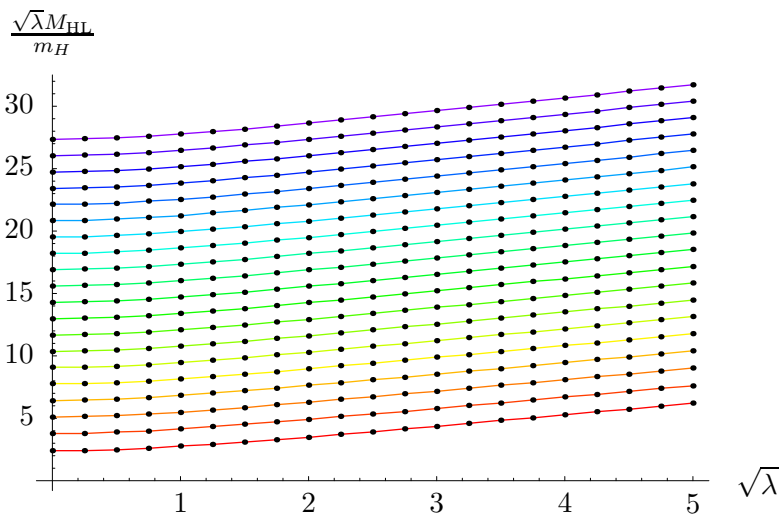


Figure 4: A plot of M_{HL}/m_H as function of the 't Hooft coupling λ for small 'tHooft coupling (the light quark mass is zero) in AdS. For small values of λ , one obtains $M_{\text{HL}}/m_H \sim \text{const.}/\sqrt{\lambda} + \mathcal{O}(\lambda)$. Note however that the supergravity background is unreliable in this limit.

and the dilaton and four-form are given by

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^\Delta, \quad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz. \quad (3.3)$$

There are formally two free parameters, R and b , since

$$\delta = \frac{R^4}{2b^4}, \quad \Delta^2 = 10 - \delta^2. \quad (3.4)$$

We see that b has length dimension one and enters to the fourth power. The $\text{SO}(6)$ symmetry of the geometry is retained at all values of the radius, w . We conclude that in the field theory a dimension four operator with no $\text{SO}(6)$ charge has been switched on. $b^4/(2\pi\alpha')^4$ therefore corresponds to a VEV for the operator $\text{Tr } F^2$. Since b is the only conformal symmetry breaking parameter in the theory it sets the intrinsic scale of strong coupling in terms of $\Lambda_b = b/(2\pi\alpha')$. In order to present numerical results below, we will set $b = R$ which corresponds to $\delta = 1/2$ — this is a representative point in the parameter space that displays chiral symmetry breaking.

Quarks are again introduced by including probe D7 branes into the geometry. Substituting into the DBI action (2.2) for this geometry we find [4] the equation of motion for the radial separation, v , of the two branes in the 8,9 directions as a function of the radial

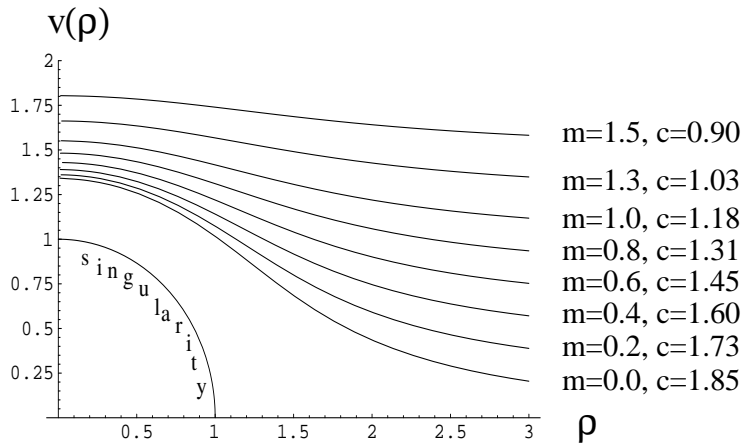


Figure 5: A plot of the embedding of the D7 brane as a function of the radial coordinate ρ .

coordinate $\rho^2 = \sum_{i=1}^4 w_i^2$ in the 4 – 7 directions,

$$\frac{d}{d\rho} \left[\frac{e^\phi \mathcal{G}(\rho, v)}{\sqrt{1 + (\partial_\rho v)^2}} (\partial_\rho v) \right] - \sqrt{1 + \partial_\rho v^2} \frac{d}{dv} \left[e^\phi \mathcal{G}(\rho, v) \right] = 0, \quad (3.5)$$

where

$$\mathcal{G}(\rho, v) = \rho^3 \frac{((\rho^2 + v^2)^2 + b)((\rho^2 + v^2)^2 - b)}{(\rho^2 + v^2)^4}. \quad (3.6)$$

At large ρ the solutions take the form

$$v = m + \frac{c}{\rho^2} + \dots \quad (3.7)$$

There are two free parameters in the solution. The first, m , describes the asymptotic separation of the D3 and D7 branes and has conformal dimension one — it is the bare quark mass. The second parameter, c , has dimension three and corresponds to the $\bar{\psi}\psi$ quark bilinear. To obtain solutions which are regular in the IR, we impose the boundary condition $\dot{v}(0) = 0$, where the dot indicates a ρ derivative, as well as fixing m in the UV. Regular solutions are displayed in figure 5. The regularity condition fixes the condensate c as a function of the quark mass m .

The solutions show that a dynamical mass is formed for the quarks. A massless quark would correspond to a D7 brane that intersects the D3 brane, such that there was a zero length string between them. We see that the D3's repel the D7 and for all configurations there is a non-zero minimum length string. The solution which asymptotically has $m = 0$ also explicitly breaks the U(1) symmetry in the 8,9 plane by bending off the axis. This is the geometric representation of the spontaneous breaking of the U(1) axial symmetry of the quarks.

Fluctuations of the brane about the solution found above in the 8,9 directions correspond to excitations of the operator $\bar{\psi}\psi$ and contain information about the pion and sigma

field of the model. With $w_6 + iw_5 = v(\rho) U(\rho, x)$ and expanding to second order in $U(\rho, x)$, the DBI action (2.2) becomes

$$S = -T_7 \int d^8\xi e^\phi \sqrt{-g}(1 + \dot{v}^2)^{\frac{1}{2}} \left[1 + \frac{1}{2} g_{\rho\rho} v^2 (1 + \dot{v}^2)^{-1} \partial^a U \partial_a U^\dagger \right]. \quad (3.8)$$

With $U(\rho, x) = \exp(i\pi(\rho, x))$, this gives an action for the pion field π and for the sigma field (here denoted by v).

There is also a superpartner $U(1)$ gauge field in the action which describes the operator $\bar{\psi}\gamma^\mu\psi$ and hence vector mesons. This is introduced as a gauge field F_{ab} living on the D7, such that the DBI action now reads

$$S_F = -T_7 \int d^8\xi e^\phi \left[-\det(\mathbf{P}[g_{ab}] + 2\pi\alpha' e^{-\phi/2} F_{ab}) \right]^{\frac{1}{2}}, \quad (3.9)$$

which, expanded to second order, gives

$$S_F = -T_7 \int d^8\xi e^\phi \sqrt{-g}(1 + \dot{v}^2)^{\frac{1}{2}} \left[1 + \frac{1}{2} g_{\rho\rho} v^2 (1 + \dot{v}^2)^{-1} \partial^a U \partial_a U^\dagger - \frac{1}{4} (2\pi\alpha')^2 e^{-\phi} F^2 \right]. \quad (3.10)$$

The pseudo-scalar mesons, i.e. the pions, correspond to fluctuations in the position of the brane in the angular direction in the $w_5 - w_6$ plane (we have suppressed fluctuations of the radial field v for simplicity). In this paper we use the predictions for the vector meson masses. They correspond to regular solutions of the equation of motion for the gauge potential A_μ . The equation of motion for a solution of the form $A_\mu = V_\mu(\rho) e^{iq \cdot x}$, $M^2 = -q^2$ is (we use the scaled coordinate $\hat{\rho} = \rho/b$, $\hat{v} = v/b$)

$$\partial_{\hat{\rho}}(K_1(\hat{\rho})\partial_{\hat{\rho}}V_\mu(\hat{\rho})) - b^2 q^2 K_2(\hat{\rho})V_\mu(\hat{\rho}) = 0, \quad (3.11)$$

with

$$K_1 = X^{1/2} Y \hat{\rho}^3 (1 + \hat{v}^2)^{-1/2}, \quad K_2 = H X^{1-\delta/2} Y^2 \hat{\rho}^3 (1 + \hat{v}^2)^{-1/2},$$

and

$$X = \frac{(\hat{v}^2 + \hat{\rho}^2)^2 + 1}{(\hat{v}^2 + \hat{\rho}^2)^2 - 1}, \quad Y = \frac{(\hat{v}^2 + \hat{\rho}^2)^2 - 1}{(\hat{v}^2 + \hat{\rho}^2)^2}. \quad (3.12)$$

The boundary conditions imply $V^\mu \sim 1/\hat{\rho}^2$ at large $\hat{\rho}$. Note that in the coordinates we are using, (3.11) provides us with the quantity $bM_{\text{vec}} = RM_{\text{vec}}/(2\delta)^{1/4}$, with M_{vec} the meson mass, as a function of the field theory quantities $m_{\text{quark}}/\Lambda_b$ with $\Lambda_b = bT$ being the effective QCD scale.

3.2 Heavy-light mesons

The formalism for studying heavy-light mesons as presented in section 2.2 moves across almost directly from the AdS geometry to the dilaton flow case. Since the five-sphere is undeformed, the action for a string tied between two D7 branes again takes the form

$$S_P = -\frac{T}{2} \int d\tau \left[-f(\rho)\dot{x}^2 - g(\rho)\dot{w}_i^2 + g(\rho) \right] \quad (3.13)$$

as in (2.12), where now

$$\begin{aligned}
 f(\rho) &= \int_{v(m_l)}^{v(m_h)} dv e^{\phi/2} g_{xx} = \int_{v(m_l)}^{v(m_h)} dv (X^{1/2} - 1)^{-1/2} X^{(\Delta+\delta)/4}, \\
 g(\rho) &= \int_{v(m_l)}^{v(m_h)} dv e^{\phi/2} g_{ww} = \int_{v(m_l)}^{v(m_h)} dv (1 - b^4(v^2 + \rho^2)^{-2})(X^\delta - 1)^{1/2} X^{(2+\Delta-\delta)/4},
 \end{aligned}
 \tag{3.14}$$

with X defined in (3.12). The integration limits for the v integrals, $v(m_h)$ and $v(m_l)$, are given by the D7 probe embeddings found above, as shown in 5. Since we only know those embeddings numerically, f and g are also only known numerically. Nevertheless we find the regular solutions of the wave equation

$$\left[\square_x^2 + \frac{f(\rho)}{g(\rho)} \nabla_w^2 - T^2 f(\rho)g(\rho) \right] \phi = 0,
 \tag{3.15}$$

again using numerics. The UV limit coincides with the AdS model with solutions $\phi = m_{\text{HL}} + \frac{c_{\text{HL}}}{\rho^2} + \dots$. We take $m_{\text{HL}}, c_{\text{HL}} = 0$ since such mass mixing and condensate terms are absent in QCD. Then we again seek linearized fluctuations of the form

$$\phi = h(\rho)e^{ik \cdot x}, \quad M^2 = -k^2,
 \tag{3.16}$$

where the large ρ behaviour of h is $1/\rho^2$. To do so it is again convenient to work in rescaled coordinates, $\hat{\rho} = \rho/b$ and $\hat{v} = v/b$, such that equation (3.15) reads

$$\left[\frac{2\pi\delta}{\lambda} \hat{f}/\hat{g} \nabla_{\hat{\rho}}^2 + \frac{M^2}{\Lambda_b^2} - \hat{f}\hat{g} \right] \phi(\hat{\rho}) = 0,
 \tag{3.17}$$

where $\Lambda_b = bT$ is the QCD scale; while \hat{f} and \hat{g} are dimensionless quantities that can be effectively obtained from (3.14) by setting $b = 1$. In these coordinates the heavy quark mass enters through the integrations limits in (3.14) in the combination m_H/Λ_b .

The masses of the heavy-light mesons are very similar in this model to those in AdS. To make the deviations clear we plot the binding energy $(M_{\text{HL}} - m_H)/m_H$. This quantity is shown in figure 6 as a function of the heavy quark mass (with massless light quark) and at fixed 't Hooft coupling ($\sqrt{\lambda} = 100$). We find the same basic behaviour for all λ . When m_H/Λ_b is large, the model returns to AdS-like results. As the heavy quark mass comes down towards the scale of the chiral symmetry breaking Λ_b , there are larger deviations as one would expect. Note that at large m_H the binding energy asymptotes to a constant - this means that since all the excited states have a mass which is to first order just m_H , the percentage difference in the masses of these states tends to zero in this limit.

In figure 7 the binding energy is shown for a fixed value of $m_H/\Lambda_b = 12.63$ as a function of λ . Again the precise value of m_H is not important here (though it will be below). At large λ we again see convergence to the AdS results - in this limit the UV of the theory is very strongly coupled and the growth of the coupling near $r = b$ (where it diverges) is presumably less important than at small λ , where the strongly coupled IR behaviour takes more prominence. The higher excited states feel the effects of the IR dynamics more strongly possibly because these states are larger.

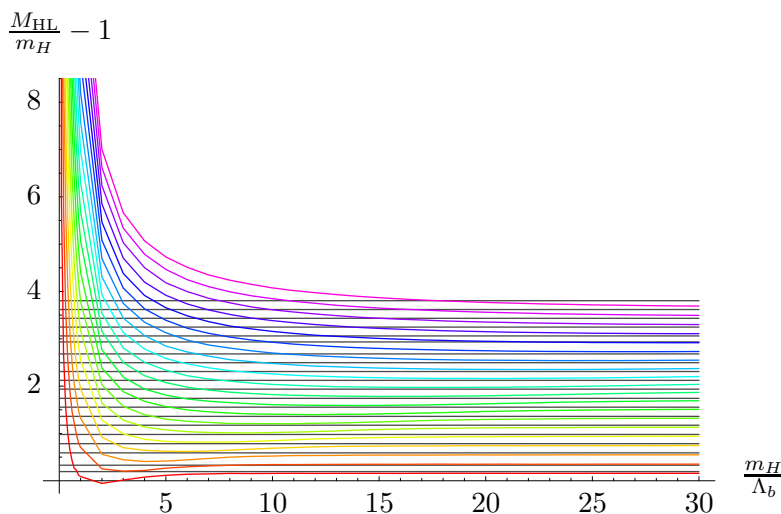


Figure 6: Binding energy as a function of the ratio heavy quark mass over QCD scale for the Constable-Myers background, for 't Hooft coupling $\lambda = 10^4$. For large ratio, the AdS behaviour (shown in black) is approached, whereas for small ratio effects of the chiral symmetry breaking are seen.

4. Bottom phenomenology

Recently there have been a number of attempts to convert these stringy holographic descriptions of chiral symmetry breaking into phenomenological models of QCD. In particular, models involving a simple AdS slice [24, 25], or the dilaton flow geometry described above [33], have proven to be successful, giving agreement to light quark meson data at better than the 20% level. In many ways this good agreement with experiment is surprising since these models become strongly coupled conformal theories in the ultra-violet rather than asymptotically free. Encouraged by the success of those models though let us consider comparing our dilaton flow model, that incorporates chiral symmetry breaking, to the bottom quark sector of the QCD spectrum. The D7 brane generating the bottom quark will lie in the AdS-like regime of our model. In QCD the dynamics should be perturbative at this energy scale but this clearly can not be achieved in a gravity dual. We hope that AdS is the next best scenario since the quarks lie in the conformal gauge background of the supersymmetric theory where strong non-renormalization theorems apply, in some way mimicking a perturbative regime.

In fact the models discussed in this paper appear somewhat different from QCD. The masses of the heavy-heavy and light-light meson states are suppressed relative to the quark mass by a factor of the 't Hooft coupling (see (2.8)) which according to the AdS/CFT Maldacena limit is big. On the other hand, the heavy-light meson masses are

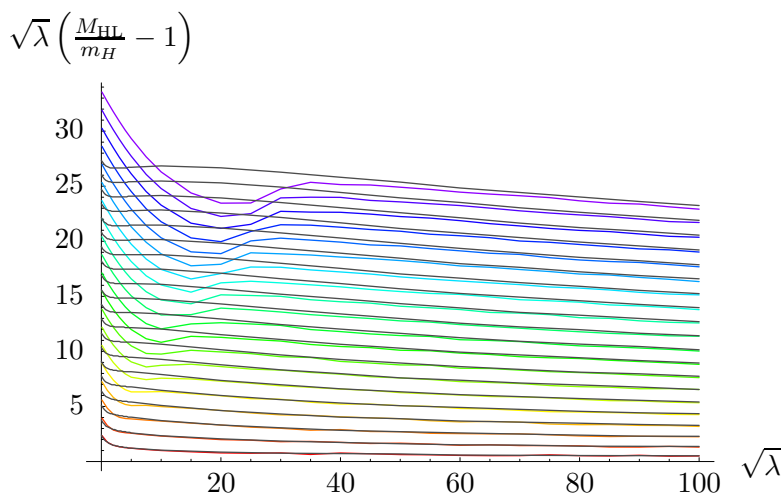


Figure 7: Binding energy for fixed $m_H/\Lambda_b = 12.63$ as a function of $\sqrt{\lambda}$ in the Constable-Myers background. The AdS behaviour is again shown in black for comparison.

not suppressed by λ (see Fig 3). A large parametric suppression of this form is not apparent in the QCD data for the bottom quark although the Υ mass is less than twice the B meson mass ($m_\Upsilon = 1.8M_B$).

Nevertheless we can attempt phenomenology. We use the equations for the vector meson masses, M_{vec} , in the Constable-Myers geometry (3.11) to determine the value of the heavy (bottom) quark mass. In fact that equation gives bM_{vec} as a function of M_H/Λ_b , so by engineering the correct ratio of the Υ to ρ meson masses (we assume the light quarks are massless) we find $m_H/\Lambda_b = 12.63$. We then substitute this value of m_H into the equations for the heavy-light meson mass (3.17). We still have the value of the 't Hooft parameter to fix which we can determine from requiring that we obtain the correct B-meson mass.

We can determine λ from the physical value of the B and ρ quark ratio using

$$\left(\frac{M_B}{M_\rho}\right)^{\text{phys}} = \frac{M_{HL}}{\Lambda_b} \frac{\Lambda_b}{M_{vec,LL}} = \frac{M_{HL}}{\Lambda_b} \frac{1}{bM_{vec,LL}} \sqrt{\frac{2\pi\delta}{\lambda}}. \quad (4.1)$$

Calculating M_{HL}/Λ_b as explained in section 3.2 for $m_H/\Lambda_b = 12.63$, the relation (4.1) allows us to determine λ . We find $\lambda = 5.22$. Whilst this value is not very large it hopefully is sufficiently big to make use of our large λ approximation. Moreover, evaluating M_{HL}/Λ_b at $\lambda = 5.22$, and identifying M_{HL} with the physical B quark mass $M_B = 5279$ MeV, we find for the QCD scale $\Lambda_b = 340$ MeV, which is a little too high compared with QCD expectations, though of the right order of magnitude. We also find $m_H = 12.63\Lambda_b = 4294$ MeV for the physical b quark mass.

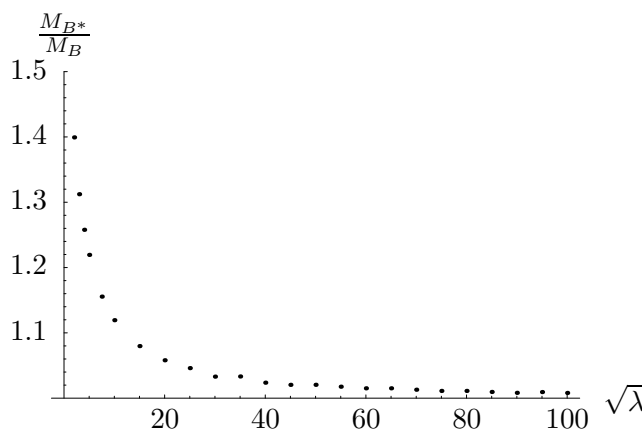


Figure 8: Ratio of the masses of the first excited and lowest heavy-light states as a function of the 't Hooft coupling. From experimental data we expect $M_{B^*}/M_B \approx 1.01$. In our model, this ratio converges to one for infinite λ .

We can now predict the excited B-meson masses. We plot M_{B^*}/M_B against the 't Hooft coupling for $m_H/\Lambda_b = 12.63$ in figure 8. Formally we have in the spirit of (4.1)

$$M_{B^*}^{\text{phys}} = M_{\rho}^{\text{phys}} \frac{M_B}{\Lambda_b} \frac{1}{bM_{\rho}} \sqrt{\frac{2\pi\delta}{\lambda}}, \tag{4.2}$$

with all quantities on the right again computable at a given λ . At $\lambda = 5.2$ we predict $M_{B^*} = 6403 \text{ MeV}$ which is 20% larger than the measured value of 5325 MeV . To approach the physical M_{B^*}/M_B ratio (which is 1.01) would require a much larger value of λ as can be seen in figure 8 - but in this case, the light-light and heavy-heavy vector meson masses would then be overly suppressed.³

In conclusion, although the model is not a perfect description of QCD, it does display the approximate pattern of the QCD b-quark mesons if we take a moderate value of the 't Hooft coupling.

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³For comparison with QCD phenomenology, we note that there are flux tube and QCD string models which allow the calculation of the B mass from the ρ and Υ masses, for instance [75, 76].

References

- [1] J.M. Maldacena, *The large- N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)].
- [2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [[hep-th/9802109](#)].
- [3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)].
- [4] J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [[hep-th/0306018](#)].
- [5] N.J. Evans and J.P. Shock, *Chiral dynamics from AdS space*, *Phys. Rev.* **D 70** (2004) 046002 [[hep-th/0403279](#)].
- [6] N. Evans, J.P. Shock and T. Waterson, *D7 brane embeddings and chiral symmetry breaking*, *JHEP* **03** (2005) 005 [[hep-th/0502091](#)].
- [7] R. Apreda, J. Erdmenger and N. Evans, *Scalar effective potential for D7 brane probes which break chiral symmetry*, *JHEP* **05** (2006) 011 [[hep-th/0509219](#)].
- [8] R. Apreda, J. Erdmenger, N. Evans and Z. Guralnik, *Strong coupling effective Higgs potential and a first order thermal phase transition from AdS/CFT duality*, *Phys. Rev.* **D 71** (2005) 126002 [[hep-th/0504151](#)].
- [9] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Towards a holographic dual of large- N_c QCD*, *JHEP* **05** (2004) 041 [[hep-th/0311270](#)].
- [10] J.L.F. Barbon, C. Hoyos, D. Mateos and R.C. Myers, *The holographic life of the eta'*, *JHEP* **10** (2004) 029 [[hep-th/0404260](#)].
- [11] K. Ghoroku and M. Yahiro, *Chiral symmetry breaking driven by dilaton*, *Phys. Lett.* **B 604** (2004) 235 [[hep-th/0408040](#)].
- [12] I. Brevik, K. Ghoroku and A. Nakamura, *Meson mass and confinement force driven by dilaton*, *Int. J. Mod. Phys.* **D15** (2006) 57 [[hep-th/0505057](#)].
- [13] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, *Prog. Theor. Phys.* **113** (2005) 843 [[hep-th/0412141](#)].
- [14] T. Sakai and S. Sugimoto, *More on a holographic dual of QCD*, *Prog. Theor. Phys.* **114** (2006) 1083 [[hep-th/0507073](#)].
- [15] E. Antonyan, J.A. Harvey, S. Jensen and D. Kutasov, *NJL and QCD from string theory*, [hep-th/0604017](#).
- [16] D. Bak and H.-U. Yee, *Separation of spontaneous chiral symmetry breaking and confinement via AdS/CFT correspondence*, *Phys. Rev.* **D 71** (2005) 046003 [[hep-th/0412170](#)].
- [17] K. Ghoroku, T. Sakaguchi, N. Uekusa and M. Yahiro, *Flavor quark at high temperature from a holographic model*, *Phys. Rev.* **D 71** (2005) 106002 [[hep-th/0502088](#)].
- [18] D. Mateos, R.C. Myers and R.M. Thomson, *Holographic phase transitions with fundamental matter*, *Phys. Rev. Lett.* **97** (2006) 091601 [[hep-th/0605046](#)].

- [19] T. Albash, V. Filev, C.V. Johnson and A. Kundu, *A topology-changing phase transition and the dynamics of flavour*, [hep-th/0605088](#).
- [20] T. Albash, V. Filev, C.V. Johnson and A. Kundu, *Global currents, phase transitions and chiral symmetry breaking in large- N_c gauge theory*, [hep-th/0605175](#).
- [21] A. Parnachev and D.A. Sahakyan, *Chiral phase transition from string theory*, *Phys. Rev. Lett.* **97** (2006) 111601 [[hep-th/0604173](#)].
- [22] O. Aharony, J. Sonnenschein and S. Yankielowicz, *A holographic model of deconfinement and chiral symmetry restoration*, [hep-th/0604161](#).
- [23] A. Karch and A. O'Bannon, *Chiral transition of $N = 4$ super Yang-Mills with flavor on a 3-sphere*, *Phys. Rev. D* **74** (2006) 085033 [[hep-th/0605120](#)].
- [24] J. Erlich, E. Katz, D.T. Son and M.A. Stephanov, *QCD and a holographic model of hadrons*, *Phys. Rev. Lett.* **95** (2005) 261602 [[hep-ph/0501128](#)].
- [25] L. Da Rold and A. Pomarol, *Chiral symmetry breaking from five dimensional spaces*, *Nucl. Phys. B* **721** (2005) 79 [[hep-ph/0501218](#)].
- [26] G.F. de Teramond and S.J. Brodsky, *The hadronic spectrum of a holographic dual of QCD*, *Phys. Rev. Lett.* **94** (2005) 201601 [[hep-th/0501022](#)].
- [27] S.J. Brodsky and G.F. de Teramond, *Hadronic spectra and light-front wavefunctions in holographic QCD*, *Phys. Rev. Lett.* **96** (2006) 201601 [[hep-ph/0602252](#)].
- [28] J. Hirn and V. Sanz, *Interpolating between low and high energy QCD via a 5d Yang-Mills model*, *JHEP* **12** (2005) 030 [[hep-ph/0507049](#)].
- [29] J. Hirn, N. Rius and V. Sanz, *Geometric approach to condensates in holographic QCD*, *Phys. Rev. D* **73** (2006) 085005 [[hep-ph/0512240](#)].
- [30] H. Boschi-Filho and N.R.F. Braga, *QCD/string holographic mapping and glueball mass spectrum*, *Eur. Phys. J. C* **32** (2004) 529 [[hep-th/0209080](#)].
- [31] H. Boschi-Filho and N.R.F. Braga, *Gauge/string duality and scalar glueball mass ratios*, *JHEP* **05** (2003) 009 [[hep-th/0212207](#)].
- [32] S. Hong, S. Yoon and M.J. Strassler, *Quarkonium from the fifth dimension*, *JHEP* **04** (2004) 046 [[hep-th/0312071](#)].
- [33] N. Evans and T. Waterson, *Improving the infra-red of holographic descriptions of QCD*, [hep-ph/0603249](#).
- [34] L. Da Rold and A. Pomarol, *The scalar and pseudoscalar sector in a five-dimensional approach to chiral symmetry breaking*, *JHEP* **01** (2006) 157 [[hep-ph/0510268](#)].
- [35] J.P. Shock, *Canonical coordinates and meson spectra for scalar deformed $N = 4$ SYM from the AdS/CFT correspondence*, *JHEP* **10** (2006) 043 [[hep-th/0601025](#)].
- [36] J.P. Shock and F. Wu, *Three flavour QCD from the holographic principle*, *JHEP* **08** (2006) 023 [[hep-ph/0603142](#)].
- [37] K. Ghoroku and M. Yahiro, *Holographic model for mesons at finite temperature*, *Phys. Rev. D* **73** (2006) 125010 [[hep-ph/0512289](#)].
- [38] K. Ghoroku, N. Maru, M. Tachibana and M. Yahiro, *Holographic model for hadrons in deformed AdS_5 background*, *Phys. Lett. B* **633** (2006) 602 [[hep-ph/0510334](#)].

- [39] K. Ghoroku, A. Nakamura and M. Yahiro, *Holographic model at finite temperature with R-charge density*, *Phys. Lett. B* **638** (2006) 382 [[hep-ph/0605026](#)].
- [40] A. Karch, E. Katz, D.T. Son and M.A. Stephanov, *Linear confinement and AdS/QCD*, *Phys. Rev. D* **74** (2006) 015005 [[hep-ph/0602229](#)].
- [41] O. Andreev, *$1/Q^2$ corrections and gauge/string duality*, *Phys. Rev. D* **73** (2006) 107901 [[hep-th/0603170](#)].
- [42] O. Andreev and V.I. Zakharov, *Heavy-quark potentials and AdS/QCD*, *Phys. Rev. D* **74** (2006) 025023 [[hep-ph/0604204](#)].
- [43] T. Hambye, B. Hassanain, J. March-Russell and M. Schwelling, *On the $\Delta_I = 1/2$ rule in holographic QCD*, *Phys. Rev. D* **74** (2006) 026003 [[hep-ph/0512089](#)].
- [44] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [[hep-th/0205236](#)].
- [45] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda and R. Marotta, *$4N = 2$ gauge theories on systems of fractional D3/D7 branes*, *Nucl. Phys. B* **621** (2002) 157 [[hep-th/0107057](#)].
- [46] M. Graña and J. Polchinski, *Gauge/gravity duals with holomorphic dilaton*, *Phys. Rev. D* **65** (2002) 126005 [[hep-th/0106014](#)].
- [47] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *JHEP* **07** (2003) 049 [[hep-th/0304032](#)].
- [48] R.C. Myers and R.M. Thomson, *Holographic mesons in various dimensions*, *JHEP* **09** (2006) 066 [[hep-th/0605017](#)].
- [49] T. Sakai and J. Sonnenschein, *Probing flavored mesons of confining gauge theories by supergravity*, *JHEP* **09** (2003) 047 [[hep-th/0305049](#)].
- [50] J. Erdmenger, J. Große and Z. Guralnik, *Spectral flow on the Higgs branch and AdS/CFT duality*, *JHEP* **06** (2005) 052 [[hep-th/0502224](#)].
- [51] I. Kirsch and D. Vaman, *The $d3/d7$ background and flavor dependence of Regge trajectories*, *Phys. Rev. D* **72** (2005) 026007 [[hep-th/0505164](#)].
- [52] D. Arean and A.V. Ramallo, *Open string modes at brane intersections*, *JHEP* **04** (2006) 037 [[hep-th/0602174](#)].
- [53] C. Núñez, A. Paredes and A.V. Ramallo, *Flavoring the gravity dual of $N = 1$ Yang-Mills with probes*, *JHEP* **12** (2003) 024 [[hep-th/0311201](#)].
- [54] X.-J. Wang and S. Hu, *Intersecting branes and adding flavors to the Maldacena- Núñez background*, *JHEP* **09** (2003) 017 [[hep-th/0307218](#)].
- [55] J. Erdmenger and I. Kirsch, *Mesons in gauge/gravity dual with large number of fundamental fields*, *JHEP* **12** (2004) 025 [[hep-th/0408113](#)].
- [56] R. Casero, C. Núñez and A. Paredes, *Towards the string dual of $N = 1$ SQCD-like theories*, *Phys. Rev. D* **73** (2006) 086005 [[hep-th/0602027](#)].
- [57] J.D. Edelstein and R. Portugues, *Gauge/string duality in confining theories*, *Fortschr. Phys.* **54** (2006) 525 [[hep-th/0602021](#)].
- [58] R. Casero, A. Paredes and J. Sonnenschein, *Fundamental matter, meson spectroscopy and non-critical string/gauge duality*, *JHEP* **01** (2006) 127 [[hep-th/0510110](#)].

- [59] T. Hirayama, *A holographic dual of CFT with flavor on de Sitter space*, *JHEP* **06** (2006) 013 [[hep-th/0602258](#)].
- [60] R.A. Janik and R. Peschanski, *Asymptotic perfect fluid dynamics as a consequence of AdS/CFT*, *Phys. Rev. D* **73** (2006) 045013 [[hep-th/0512162](#)].
- [61] F. Canoura, J.D. Edelstein, L.A.P. Zayas, A.V. Ramallo and D. Vaman, *Supersymmetric branes on $AdS_5 \times Y^{p,q}$ and their field theory duals*, *JHEP* **03** (2006) 101 [[hep-th/0512087](#)].
- [62] S. Benvenuti, M. Mahato, L.A. Pando Zayas and Y. Tachikawa, *The gauge / gravity theory of blown up four cycles*, [hep-th/0512061](#).
- [63] A. Paredes and P. Talavera, *Multiflavour excited mesons from the fifth dimension*, *Nucl. Phys. B* **713** (2005) 438 [[hep-th/0412260](#)].
- [64] J.M. Pons, J.G. Russo and P. Talavera, *Semiclassical string spectrum in a string model dual to large- N QCD*, *Nucl. Phys. B* **700** (2004) 71 [[hep-th/0406266](#)].
- [65] K. Peeters, J. Sonnenschein and M. Zamaklar, *Holographic decays of large-spin mesons*, *JHEP* **02** (2006) 009 [[hep-th/0511044](#)].
- [66] A.L. Cotrone, L. Martucci and W. Troost, *String splitting and strong coupling meson decay*, *Phys. Rev. Lett.* **96** (2006) 141601 [[hep-th/0511045](#)].
- [67] A.L. Cotrone, L. Martucci, J.M. Pons and P. Talavera, *Heavy hadron spectra from spin chains and strings*, [hep-th/0604051](#).
- [68] M. Bando, A. Sugamoto and S. Terunuma, *Meson strings and flavor branes*, *Prog. Theor. Phys.* **115** (2006) 1111 [[hep-ph/0602203](#)].
- [69] C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L.G. Yaffe, *Energy loss of a heavy quark moving through $N = 4$ supersymmetric Yang-Mills plasma*, *JHEP* **07** (2006) 013 [[hep-th/0605158](#)].
- [70] A.V. Manohar and M.B. Wise, *Heavy quark physics*, *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **10** (2000) 1.
- [71] N.R. Constable and R.C. Myers, *Exotic scalar states in the AdS/CFT correspondence*, *JHEP* **11** (1999) 020 [[hep-th/9905081](#)].
- [72] S.S. Gubser, *Dilaton-driven confinement*, [hep-th/9902155](#).
- [73] A. Kehagias and K. Sfetsos, *On running couplings in gauge theories from type-IIB supergravity*, *Phys. Lett. B* **454** (1999) 270 [[hep-th/9902125](#)].
- [74] V. Borokhov and S.S. Gubser, *Non-supersymmetric deformations of the dual of a confining gauge theory*, *JHEP* **05** (2003) 034 [[hep-th/0206098](#)].
- [75] A.B. Kaidalov, *Hadronic mass relations from topological expansion and string model*, *Zeit. Phys.* **C12** (1982) 63.
- [76] Y.A. Simonov, *Heavy - light mesons and quark constants $f(B)$, $f(D)$ in the method of vacuum correlators*, *Z. Physik C* **53** (1992) 419.